Modulation

Lesson 13

Sec 1.1.6

Homework

- Problem (1)
  - The signal $f(t)=(1+m_o\cos t)\cos 100t$ is applied to a series RLC circuit where L=10 h, C=10 μF, R=10k Ω. Calculate *v(t),* the voltage across the resistance.
- Problem (2)
  - The SSBSC version of *f(t)* is applied to the same circuit. Calculate *v(t)*.

Homework Answers #1

- Problem (1)
  - The signal $f(t)=(1+m_o cos\ t)cos100t$ is applied to a series RLC circuit where L=10 h, C=10 μF, R=10k Ω. Calculate $v(t)$, the voltage across the resistance.

Recall:

$$\Im[f(t)] = \Im[K\{1+m(t)\}\cos\omega_o t]$$

$$= K\pi\{\delta(\omega-\omega_o)+\delta(\omega+\omega_o)\}+\frac{K}{2}\{M[j(\omega-\omega_o)]+M[j(\omega+\omega_o)]\}$$

$$=\pi\{\delta(\omega-100)+\delta(\omega+100)\}+\frac{1}{2}\{M[j(\omega-100)]+M[j(\omega+100)]\}$$

since $m(t) = \cos t$

then $\Im[m(t)] = \pi[\delta(\omega-1)+\delta(\omega+1)]$

$$\Im[f(t)] = F(j\omega) = \pi\{\delta(\omega-100)+\delta(\omega+100)\}+\frac{m_o}{2}\{\pi[\delta(\omega-1-100)+\delta(\omega+1-100)]+\pi[\delta(\omega-1+100)+\delta(\omega+1+100)]\}$$

$$=\pi[\delta(\omega-100)+\delta(\omega+100)+\frac{m_o}{2}\{[\delta(\omega-101)+\delta(\omega-99)]+[\delta(\omega+99)+\delta(\omega+101)]\}]$$

Homework Answers #2

- Problem (1)
  - The signal $f(t)=(1+m_o cos\ t)cos100t$ is applied to a series RLC circuit where L=10 h, C=10 µF, R=10k Ω. Calculate $v(t)$, the voltage across the resistance.

$$H(j\omega)=\frac{R}{R+j\omega L+\frac{1}{j\omega C}}=\frac{10k}{10k+j\omega 10+\frac{1}{j\omega 10\times 10^{-6}}}$$

$$=\frac{j\omega 10\times 10^{-6}\times 10k}{j\omega 10\times 10^{-6}\times 10k+j\omega 10\times 10^{-6}\times j\omega 10+1}$$

$$=\frac{j\omega 10^{-1}}{1-\omega^2 10^{-4}+j\omega 10^{-1}}$$

$$H(j\omega)=\frac{j\omega 10^{-1}}{1-\omega^2 10^{-4}+j\omega 10^{-1}}$$

$$=\frac{\omega 10^{-1}}{\sqrt{(1-\omega^2 10^{-4})^2+(\omega 10^{-1})^2}}\angle 90-\tan^{-1}[\frac{\omega 10^{-1}}{(1-\omega^2 10^{-4})}]$$

$$F(j\omega)=\pi[\delta(\omega-100)+\delta(\omega+100)+\frac{m_o}{2}\{[\delta(\omega+99)+\delta(\omega-99)]+[\delta(\omega+101)+\delta(\omega-101)]\}]$$

However, $F(j\omega)$ has value only at $\omega=99,100,101$

Homework Answers #3

- Problem (1)
  - The signal *f(t)=(1+m$_o$cos t)cos100t* is applied to a series RLC circuit where L=10 h, C=10 μF, R=10k Ω. Calculate *v(t),* the voltage across the resistance.

$$H(j\omega)=\frac{j\omega 10^{-1}}{1-\omega^2 10^{-4}+j\omega 10^{-1}}$$

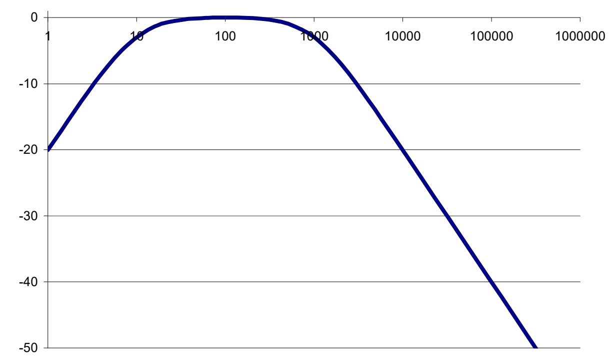
$$=\frac{\omega 10^{-1}}{\sqrt{(1-\omega^2 10^{-4})^2+(\omega 10^{-1})^2}}\angle 90-\tan^{-1}[\frac{\omega 10^{-1}}{(1-\omega^2 10^{-4})}]$$

$$H(j99)=\frac{99\times 10^{-1}}{\sqrt{(1-99^2\times 10^{-4})^2+(99\times 10^{-1})^2}}\angle 90-\tan^{-1}[\frac{99\times 10^{-1}}{(1-99^2\times 10^{-4})}]$$

$$=.9999\angle 0.115^\circ$$

$$H(j100)=1\angle 0^\circ$$

$$H(j101)=.9999\angle -0.115^\circ$$



$$F(j\omega)=\pi[\delta(\omega-100)+\delta(\omega+100)+\frac{m_o}{2}\{[\delta(\omega+99)+\delta(\omega-99)]+[\delta(\omega+101)+\delta(\omega-101)]\}]$$

$$v(t)=\cos 100t+.999m_o\cos(99t+0.115)+.999m_o\cos(101t-0.115)$$

Homework Answers #4

- Problem (2)
  - The SSBSC version of *f(t)* is applied to the same circuit. Calculate *v(t)*.

Recall:

$$\Im[f(t)] = \Im[SSBSC\{1+m_o \cos t\}\cos 100t]\} = \Im[SSBSC\{\cos 100t + \frac{m_o}{2}(\cos 101t + \cos 99t]\}$$

$$= \Im[m_o(\cos 101t)\}] = \pi\frac{m_o}{2}[\delta(\omega+101) + \delta(\omega-101)]$$

$$H(j\omega) = \frac{j\omega 10^{-1}}{1-\omega^2 10^{-4} + j\omega 10^{-1}} \qquad F(j\omega) = \pi\frac{m_o}{2}[\delta(\omega+101) + \delta(\omega-101)]$$

$$H(j101) = .9999\angle -0.115^\circ \qquad v(t) = .999 m_o \cos(101t - 0.115)$$